

Notes on Productivity

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Abstract

It is normally accepted that productivity is the key driver of growth. What could be more obvious than with increasing productivity we produce more goods and services and that makes the whole society richer and each of its member? Therefore, it is expected that members of society getting richer should receive fatter paychecks. However, the dynamics of productivity and compensation (wages and benefits) for American full-time workers for the last 16 years demonstrates that compensation grew 13% while productivity grew 38% (three times more). It would be interesting to support this empiric evidence with some theoretical construct. These NOTES demonstrate that continuous growth in productivity does not guarantee corresponding growth in public utility and that some productivity p^* exists, which is most beneficial for society as a whole. In general, the technique used in these NOTES is inspired by considerations of Laffer Curve, which shows that there some taxation rate t^* exists (between 0% and 100%) which delivers maximum for the Government Revenue. The subject matter of these NOTES –existence of an unknown optimal productivity p^* is based on considerations of productivity $p = 0$ and $p = \infty$ (infinity). Intuitively we understand the zero productivity $p = 0$ as a situation of no production/no individual earnings: a non-producing individual earns no wages and benefits. The condition of the infinite productivity $p = \infty$ requires some considerations.

1. The productivity of the whole society can be compared to the productivity of firms. There are different ways to measure productivity. It is sufficient for this analysis to accept a simple measure - Average Labor Productivity, $p = Y/N$, where Y is Gross Output and N is Labor.

As improving productivity is the central idea in companies' ongoing business development, it materializes naturally as reduction of labor N.

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Even with constant Y it leads to increasing productivity p . Ultimately, we can consider $N \rightarrow 0$ and $p \rightarrow \infty$. Certainly, this is an idealization, an abstraction, but no more outrageous than the idea of continuous compounding with an infinite number of compounding periods $n \rightarrow \infty$ and interest per compounding period $i/n \rightarrow 0$.

Also, we can fashion productivity $p \rightarrow \infty$ when $N \rightarrow 0$ after Dirak's delta function: a function on the real line which is zero everywhere except at the origin, where it is infinite:

$$\begin{cases} +\infty, & \text{when } x = 0 \\ 0, & \text{when } x \neq 0 \end{cases}$$

It should be noted, that at zero labor $N = 0$ and $p = \infty$ the product $N \cdot p = Y$. The delta function correspondingly satisfies constraint:

$$\int_{-\infty}^{\infty} \delta(x) dx = 1.$$

This constraint can be easily modified to satisfy $N \cdot p = Y$ requirement.

Summarizing, we can assert that we can accept productivity as expression $p = Y/N$ up to $N = 0$ and $p \rightarrow \infty$.

2. As was mentioned before the productivity of the whole society can be compared to the productivity of a firm. Our objective is to find out whether the objective of society to increase its productivity positively reflects on the society. Ultimately, to answer this question we can examine how an ultimate spread of productivity from 0 to ∞ reflects on some aggregate measure of society's prosperity. As such a measure we can select some abstract utility function, for example the Earned Income of the society [EI].

As we increase productivity we would expect that the earned income increases as well; otherwise, what is the sense to pursue such a goal?

So, we would examine how the earned income changes with that ultimate spread of productivity $[0, \infty)$.

It should be noted that both $p=0$ and $p \rightarrow \infty$ are idealized entities. They are constructs in some way analogous to Laffer's rates of taxation $t=0$ and $t=100\%$. For both of them, Laffer postulated corresponding tax revenue [[[The Laffer Curve: Past, Present, and Future, By Arthur Laffer; *June 1, 2004*

<http://www.heritage.org/research/reports/2004/06/the-laffer-curve-past-present-and-future>]]]:

At a tax rate of 0 percent, the government would collect no tax revenues, no matter how large the tax base.

Likewise, at a tax rate of 100 percent, the government would also collect no tax revenues because no one would willingly work for an after-tax wage of zero (i.e., there would be no tax base). Between these two extremes there are two tax rates that will collect the same amount of revenue: a high tax rate on a small tax base and a low tax rate on a large tax base.

In other words, Laffer postulates, "that no tax revenue will be raised at the extreme tax rates of 0% and 100% and that there must be at least one rate where tax revenue would be a non-zero maximum."

Both propositions about tax rates of 0% and 100% are not exact. With $t = 0$, the government as well as an organized society stop to exist. 100% taxation does not mean "no one would be willingly work." The Soviet Union economic system was based on 100% taxation as the state would expropriate the entire Gross Product, then return to the people a part of it in form of salaries and wages as personal income and then tax it at practically flat rate of 13%. It is difficult to discuss this arrangement in terms of how "willingly" was it accepted, but the society was operational for 70 years.

It is obvious that zero productivity $p=0$ reflects the society in collapse. That was Russia during the October revolution and during following several years, which are marked with word "razrukha" which means complete ruin, devastation, economic dislocation. But even during these critical years (the revolution was followed by devastating civil war) the society survived, although on bare minimum. We may accept that condition as nearest to ($p = 0$, Earned Income = 0) point.

The point $p \rightarrow \infty$ requires further review. In analogy with a firm's productivity, we obtain such idealized productivity for the whole society by accepting labor $N = 0$, which is also an idealized parameter, because it implies no wage earners/recipients, but still existing population, which does not disappear. We may envision this situation as the entire country becoming a shell, on-line based corporation having all its operations outside USA but its active market in USA. So, this situation models the entire country, the whole economy, with $p \rightarrow \infty$ having all merchandise and goods, but with $N = 0$ there is no active labor force drawing wages and salaries, or in other words there is no Earned Income.

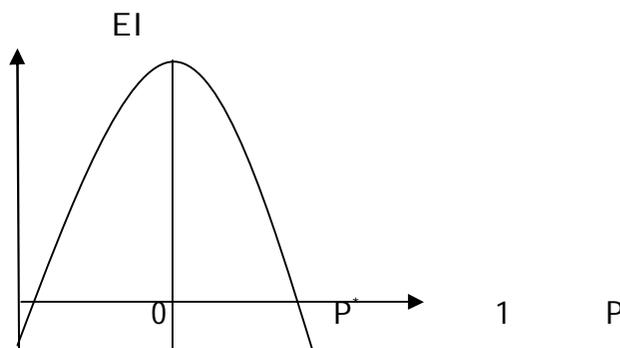
We may state, that having fruits of infinite productivity, we could just distribute them (necessities – merchandise and goods) among that existing, but not laboring ($N = 0$) population. However, our history clearly shows that during the stressed times of depressions, even with existing charities, we do not distribute all necessities among the unemployed people on unlimited basis.

Therefore we can accept another point of our considerations, that is ($p \rightarrow \infty$, $EI=0$).

Following Laffer's logic, or in more general terms, based on Rolle's theorem on existence of zero derivative on interval (a,b) when $f(a) = f(b)$, we come to the conclusion that between $p=0$ and $p \rightarrow \infty$, a productivity p^* exists, which delivers maximum to the Earned Income. That means, that increasing productivity beyond the value p^* leads to decreasing Earned Income.

The interval $p [0, \infty)$ is inconvenient for plotting and discussion. It is possible to convert interval $p [0, \infty)$ to interval $P [0,1]$ using the following transformation: $P = (e^p - 1) / e^p$. At $p = 0$ it delivers $P = 0$, and at $p \rightarrow \infty$ it delivers $P = 1$.

The above transformation allows presenting the Earned Income vs. P function as parabola open down like a curve:



On this curve P^* represents the modified productivity at which maximal Earned Income is generated. As with the Laffer curve, the P^* is not known and serves as an indication that the critical productivity exists, beyond which its increase negatively affects the Earned Income.

It should be noted as with the Laffer curve, this graph does not bear scales on both axes. Its symmetry does not bear any importance. As with the Laffer curve, it has just illustrative significance reflecting its underlying idea: that continuous growth in productivity may become damaging to the society.

1. What are the possible factors subverting the positive effect of increased productivity, which was always considered as the indispensable and crucial component of societal development, a decisive factor for accumulation of wealth, for best handling of human resources? What is the mechanism of the negative effect of increase in productivity on society at large? Here is one explanation.

The productivity increase can be achieved by different measures and accounted differently. It could be achieved just by increasing working hours by each worker. This leads to increased productivity per each worker. It could be achieved through automatization, robotization, computerization. This leads to hourly increase in productivity. In both cases the increased productivity results in layoffs and unemployment. In the ideal case, the savings from reduced labor expenses should be returned to the remaining workers in some equitable manner. The last several decades show that this did not happen.

Obviously, the society in general bears expenses to support the laid off workers and/or retrain them. It does not matter who pays for retraining: the individual, the company or the local/federal government – there will be expense anyway.

Hopefully, the new working places will be created and filled. In the case, when the rate of productivity increases and corresponding layoffs fall behind the increased rates of expenses on support and retraining, the productivity increase starts to affect negatively the society in general.

The last several decades, the American labor increased its productivity dramatically, but was negatively affected by several converging factors:

- Massive transfer of production to third world countries and especially to China,
- Massive influx of illegal labor,
- Massive retention of accumulated profits by corporations and refusal to invest and expand,
- Degrading evolution of labor to low skill/low pay jobs in retail, warehousing and services contrary to expected ascent of the whole society to high-tech/high-pay jobs.

The increased productivity not supported by corresponding increase in compensation results in decline in purchasing power of the population and as such adds another layer of problems stemming from the increased productivity

In general, the increase in productivity is directly tied to technical and technological progress. The industrial progress does not work smoothly all the time from the point of view of labor force dislocations and complications associated with it. Early 19th century's Luddism, 20th century assembly lines technology – are examples of difficulties associated with that.

It appears that in certain circumstances the increased productivity does not pay the workers and does not pay the society at large. The strains in the labor force, combined with other stressful factors - natural disasters, epidemics, wars – are breeding grounds for social unrest and revolutions.